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QCD Interference Effects of Heavy Particles Below Threshold

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ABSTRACT

We consider how two classes of heavy particles: extra vector-like families, and strongly interacting superpartners, manifest themselves below threshold, by interference of virtual loops with normal QCD processes. Quantitative estimates are presented.

1. Introduction

Although the Standard Model has proven to be remarkably successful at presently accessible energies, there is universal agreement that it is incomplete – that new structure will emerge at higher energies. The existence of new heavy particles can be inferred even at energies far below threshold if they mediate processes which violate symmetries of the low energy theory. A classic example is the weak interactions, beginning with Fermi’s theory of β decay described by a Lagrangian with symmetry breaking four fermion operators, and culminating in the discovery of W and Z. Even if it does not have the qualitative effect of violating a symmetry, exchange of virtual heavy particles can in principle generate observable quantitative consequences. This has been much discussed in the context of precision electroweak measurements [1]. Here we will briefly address another possibility, which potentially yields much larger effects, though unfortunately with more poorly controlled backgrounds. That is, we will consider the effects of interference between conventional QCD amplitudes and amplitudes involving virtual exchange of various possible heavy particles.

Given the apparent unification of couplings, which is at least in good semi-quantitative in agreement with the minimal supersymmetric Standard Model (MSSM) [2], most interest attaches to a rather short list of particles – those whose existence would not render this success coincidental. Besides the minimal superpartners, the catalog of additional particles which can appear at the TeV scale is quite short; essentially, one must consider complete SU(5) multiplets. We shall mainly focus on the superpartners in the MSSM, but we shall compare the (more easily calculated) effect of additional vectorlike families and briefly mention new Z' gauge bosons. For simplicity, we will restrict our attention to processes involving four external quarks.

2. Vacuum Polarization; Vector Families

The simplest QCD interference effects arise from vacuum polarization due to heavy particles. We write the full gluon propagator, $D'_{\mu\nu}{}^{ab}$, as

$$iD'_{\mu\nu}{}^{ab} = iD_{\mu\nu}{}^{ab} + iD_{\mu\alpha}^{am}(i\Pi_{mn}^{\alpha\beta})iD_{\beta\nu}^{nb} + \dots$$

and

$$i\Pi_{mn}^{\alpha\beta}(q) = i\delta_{mn}(q^\alpha q^\beta - g^{\alpha\beta} q^2)i\Pi(q^2) .$$

We renormalize by zero momentum subtraction; that is, by computing $\Pi(q^2) - \Pi(0)$. Well below threshold it is appropriate to expand the result in powers of q^2/m^2 , and to keep only the lowest term. For scalars of mass m_s in representation R of color SU(3) we then have:

$$\Pi(q^2) - \Pi(0) = \frac{\alpha_s}{120\pi} \frac{q^2}{m_s^2} T(R) + O\left(\frac{q^4}{m_s^4}\right) ,$$

where $T(R)$ is defined by $\text{Tr}(T_R^a T_R^b) = T(R)\delta^{ab}$. For a Dirac fermion:

$$\Pi(q^2) - \Pi(0) = \frac{\alpha_s}{15\pi} \frac{q^2}{m_f^2} T(R) + O\left(\frac{q^4}{m_f^4}\right) .$$

A Majorana or Weyl fermion contributes one half of this amount.

Interference of vacuum polarization diagrams with one gluon exchange diagrams gives the change in cross section: $\frac{\Delta\sigma}{\sigma} = -2[\Pi(q^2) - \Pi(0)]$. Therefore, n_f families of Dirac fermions contribute

$$\frac{\Delta\sigma}{\sigma} = -\frac{2\alpha_s}{15\pi} \frac{q^2}{m_f^2} n_f T(R) .$$

This formula is roughly valid until $|q^2/4m_f^2| = 1$, at which point the cross section for the dominant t and u channel processes has increased by $\Delta\sigma/\sigma \approx 2n_f T(R) \%$,

taking $\alpha_s \approx 1/8$. Strangely enough, the s channel process is actually suppressed – a warning that one must not think too naively about timelike *versus* spacelike effective couplings. For n_f vectorlike supersymmetric families ($SU(5) \bar{10}+5+10+\bar{5}$), assumed mass-degenerate, one has in the same approximations

$$\frac{\Delta\sigma}{\sigma} = -\frac{\alpha_s}{6\pi} \frac{q^2}{m_f^2} n_f T(R) \sim 5n_f\% \quad (2.1)$$

($T(R) = 4 \text{ triplets} \times \frac{1}{2} = 2$). There is of course no difficulty in using the accurate vacuum polarization formulas. This gives a slightly smaller value for the correction at $|q^2/4m| = 1$, and turns over at large q^2 into the familiar logarithmic running of the coupling.

3. Supersymmetry

In the MSSM, the particles contributing to the vacuum polarization are Majorana gluinos in the adjoint representation, and 6 flavors \times 2 chiralities = 12 squarks in the fundamental representation. So, setting all masses to be equal,

$$\Pi(q^2) - \Pi(0) = \frac{3\alpha_s}{20\pi} \frac{q^2}{m^2} .$$

However in this case vacuum polarization is not the whole story, since the presence of gluino-squark Yukawa couplings leads to additional contributions from vertex corrections and box diagrams. Supersymmetry dictates that the strength of these couplings have fixed numerical ratios to the ordinary strong coupling, and these diagrams are in no sense negligible. We have evaluated the relevant diagrams, which are displayed in Fig. 1, in Feynman gauge. In addition we have evaluated some of the diagrams corresponding to two quark - two gluon processes, which are displayed in Fig. 2. More nonvanishing diagrams can be obtained from those shown by permuting Lorentz and color indices. Although there would seem to be diagrams containing two gluon - two squark vertices which should be included with

the diagrams in Fig. 2, these turn out to vanish for on shell, massless quarks and so can be omitted. In each of the diagrams we can choose the squarks to be of either L or R type; in our results below we have added the corresponding amplitudes together. Similarly, for the Fig. 2 diagrams we have added the amplitude for the displayed diagram plus the diagram with gluon labels exchanged. For simplicity, we have given all the squarks and gluinos a common mass, m . The results for on shell, massless quarks and gluons are summarized below.

$$1a: \quad -\frac{i\alpha_s^2}{72m^2} (T_a)_{ij} (T_a)_{mn} (\gamma^\mu)_{\alpha\beta} (\gamma_\mu)_{\gamma\delta}$$

$$1b: \quad \frac{3i\alpha_s^2}{8m^2} (T_a)_{ij} (T_a)_{mn} (\gamma^\mu)_{\alpha\beta} (\gamma_\mu)_{\gamma\delta}$$

$$1c: \quad -\frac{i\alpha_s^2}{6m^2} (T_a T_b)_{in} (T_b T_a)_{mj} [(\gamma^\mu)_{\alpha\beta} (\gamma_\mu)_{\gamma\delta} + (\gamma^\mu \gamma_5)_{\alpha\beta} (\gamma_\mu \gamma_5)_{\gamma\delta} \\ - 2 (1)_{\alpha\beta} (1)_{\gamma\delta} - 2 (\gamma_5)_{\alpha\beta} (\gamma_5)_{\gamma\delta}]$$

$$1d: \quad \frac{i\alpha_s^2}{6m^2} (T_a T_b)_{ij} (T_a T_b)_{mn} [(\gamma^\mu C)_{\gamma\alpha} (C^{-1} \gamma_\mu)_{\beta\delta} + (\gamma^\mu \gamma_5 C)_{\gamma\alpha} (C^{-1} \gamma_\mu \gamma_5)_{\beta\delta} \\ - 2 (C)_{\gamma\alpha} (C^{-1})_{\beta\delta} - 2 (C \gamma_5)_{\gamma\alpha} (C^{-1} \gamma_5)_{\beta\delta}]$$

$$2a: \quad \frac{\alpha_s^2}{6m^2} f_{eac} f_{ebd} (T_c T_d)_{ij} [\gamma^\mu (3p_1^\nu - 2p_2^\nu) + (-p_1^\mu + 2p_2^\mu) \gamma^\nu \\ + \not{p}_3 \gamma^\mu \gamma^\nu - 1/2 (\gamma^\mu \not{p}_3 \gamma^\nu - \gamma^\nu \not{p}_3 \gamma^\mu)] \\ + \{ \mu \leftrightarrow \nu \quad ; \quad a \leftrightarrow b \quad ; \quad p_3 \rightarrow p_1 - p_2 - p_3 \}$$

$$2b: \quad -\frac{\alpha_s^2}{12m^2} f_{bcd} (T_c T_a T_d)_{ij} [2\gamma^\mu (p_1^\nu + p_2^\nu) - 6(p_1^\mu + p_2^\mu) \gamma^\nu + 4\not{p}_3 \gamma^\nu \gamma^\mu] \\ + \{ \mu \leftrightarrow \nu \quad ; \quad a \leftrightarrow b \quad ; \quad p_3 \rightarrow p_1 - p_2 - p_3 \}$$

$$2c: \quad \frac{i\alpha_s^2}{6m^2} (T_c T_a T_b T_c)_{ij} [\gamma^\mu (2p_1^\nu - 3p_2^\nu - p_3^\nu) + (-2p_1^\mu + p_2^\mu) \gamma^\nu - \not{p}_3 g^{\mu\nu}] \\ + \{ \mu \leftrightarrow \nu \quad ; \quad a \leftrightarrow b \quad ; \quad p_3 \rightarrow p_1 - p_2 - p_3 \}$$

$$2d: \quad \frac{\alpha_s^2}{30m^2} f_{abc} (T_c)_{ij} \left[-2\gamma^\mu p_3^\nu + 2(p_1^\mu - p_2^\mu) \gamma^\nu + \frac{p_1^\mu p_3^\nu - p_2^\mu p_3^\nu}{p_3^\sigma (p_{1\sigma} - p_{2\sigma})} \not{p}_3 + g^{\mu\nu} \not{p}_3 \right]$$

$$2e: \quad \frac{\alpha_s^2}{10m^2} f_{abc}(T_c)_{ij} \left[-8\gamma^\mu p_3^\nu + 8(p_1^\mu - p_2^\mu)\gamma^\nu - \frac{p_1^\mu p_3^\nu - p_2^\mu p_3^\nu}{p_3^\sigma(p_{1\sigma} - p_{2\sigma})} \not{p}_3 + 9g^{\mu\nu} \not{p}_3 \right]$$

f_{abc} are SU(3) structure constants, and T_a are SU(3) matrices in the fundamental representation. C is the charge conjugation operator, which arises due to the Majorana fermion propagators. When contracting with external spinors the following formulas are useful: $C\bar{u}^T(p, \pm) = v(p, \pm)$; $u^T(p, \pm)C^{-1} = -\bar{v}(p, \pm)$. In the results for the diagrams of Fig. 2, we have suppressed the spinor indices; all gamma matrices have the spinor indices $(\gamma^\mu)_{\alpha\beta}$.

In Feynman gauge the dominant contribution to four quark processes comes from the vertex correction involving a triangle with two gluinos and one squark. This diagram contributes

$$\frac{\Delta\sigma}{\sigma} = \frac{3\alpha_s}{8\pi} \frac{q^2}{m^2}.$$

Note that this comes with the opposite sign as compared to the vacuum polarization piece. Combining these, we find a small *decrease* in the cross section for t and u channel processes, of order $\Delta\sigma/\sigma \approx -1$ % at $q^2/4m^2 = -1$. Insofar as this is representative of the overall magnitude of the effect, it is disappointingly small.

Let us note that although the box diagrams are numerically smaller they do lead to a different angular distribution.

4. Comments

1. Recent CDF data on inclusive jet production in the transverse momentum region $200 < p_T < 420$ GeV indicates a cross section apparently exceeding QCD predictions by several tens of percent [3]. The results of the present paper indicate that the data cannot readily be explained by a mass threshold in the minimal supersymmetric standard model, as the main calculated effect is too small and of the wrong sign. Extra vector-like sumpersymmetric families will give a positive effect,

but still rather small unless there are many such families. After our calculations were completed, but before this note was prepared a paper by Barger, Berger, and Phillips [4] appeared, in which they claim that a mass threshold at 200 GeV, with sufficient particle content to turn off the running of α_s asymptotically, can lead to a substantial (20 %) increase in jet production at 500 GeV. They do not consider the effect of Yukawa couplings, and so their estimates might apply for generic extra strongly-interacting matter, but not for the superpartners of ordinary matter.

2. Above we have essentially considered corrections to quark-quark scattering due to virtual heavy particles; one should also consider whether exchange of such particles affects the amplitudes for finding quarks inside the initial projectiles, *i. e.* the structure functions. Are there additional contributions from this source? Thinking back to the graphical origin of structure function evolution [5], we recognize that such contributions originate from the soft side of the cut process, and thus that in calculating the effect of virtual heavy particles one would meet factors of p^2/m^2 , where p^2 is a typical hadronic momentum, making it relatively negligible.

3. A larger interference effect might be induced from exchange of an additional neutral gauge boson, say Z' . Such gluon- Z' interference is somewhat analogous to Z - γ interference below threshold in e^+e^- annihilation, with the following important difference: since Z' unlike the gluon is a color singlet, interference effects only arise from crossed channels. The analysis of this case is worthwhile but much more complicated, and will not be undertaken here.

Acknowledgments

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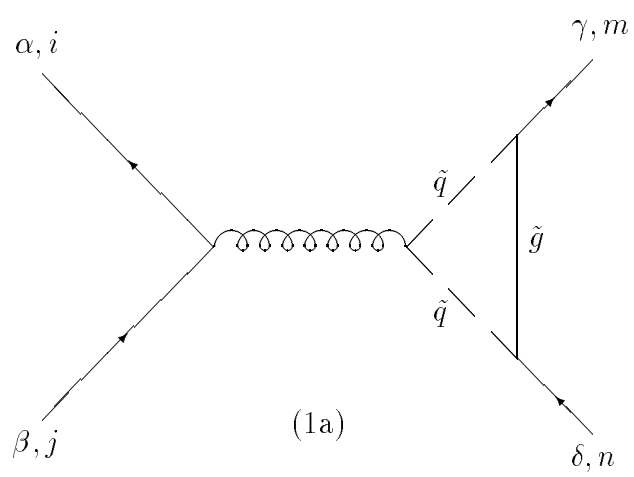
Note added 5/21/96: J. Ellis and D. Ross, in hep-ph/9604432, have significantly extended the calculations reported here. We thank them for pointing out some minor numerical errors in the original version of this note.

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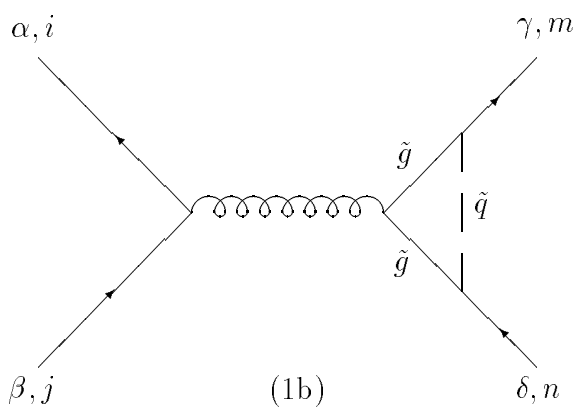
1. See F. Merritt, H. Montgomery, A. Sirlin, and M. Swartz, *Precision Tests of Electroweak Physics: Current Status and Prospects for the Next Two Decades in Particle Physics: Perspectives and Opportunities*, ed. Peccei *et al.*, (World Scientific, 1995) and references therein.
2. See S. Dimopoulos, S. Raby, and F. Wilczek, *The Unification of Couplings*, Physics Today **44**, October 1992, p. 25, and references therein.
3. A. Bhatti, *Inclusive Jet Production at Tevatron*, Rockefeller-CDF preprint /CDF/JET/PUBLIC/3229, June 1995.
4. V. Barger, M. Berger, and R. Phillips, *Thresholds in α_s Evolution and the p_T Dependence of Jets*, hep-ph/9512325, December 1995.
5. See especially S. Weinberg, *New Approach to the Renormalization Group*, Phys. Rev. **D8**, 3497 (1973).

FIGURE CAPTIONS

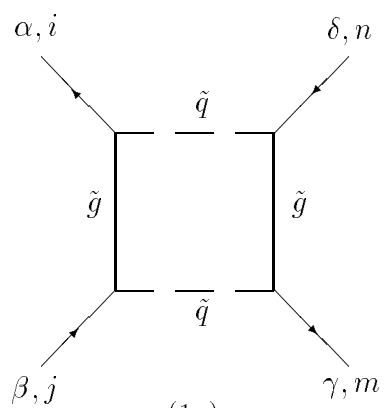
- 1) One loop diagrams contributing to four quark processes in the QCD sector of the MSSM. Lines corresponding to squarks and gluinos are labelled, as are the Lorentz and color indices of the external quarks. The remaining diagrams may be obtained from the above by permuting indices.
- 2) One loop diagrams contributing to two quark - two gluon processes. The independent external momenta are labelled, where we use the convention that positive momentum for the quarks goes in the same direction as the charge flow, and for gluons it goes into the vertex.



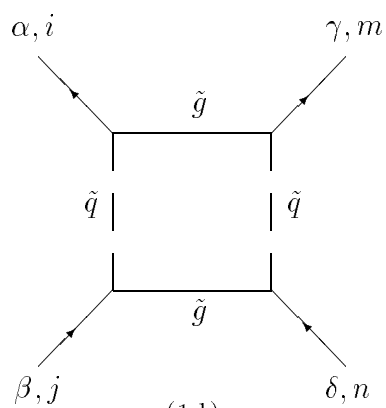
(1a)



(1b)



(1c)



(1d)

Fig. 1

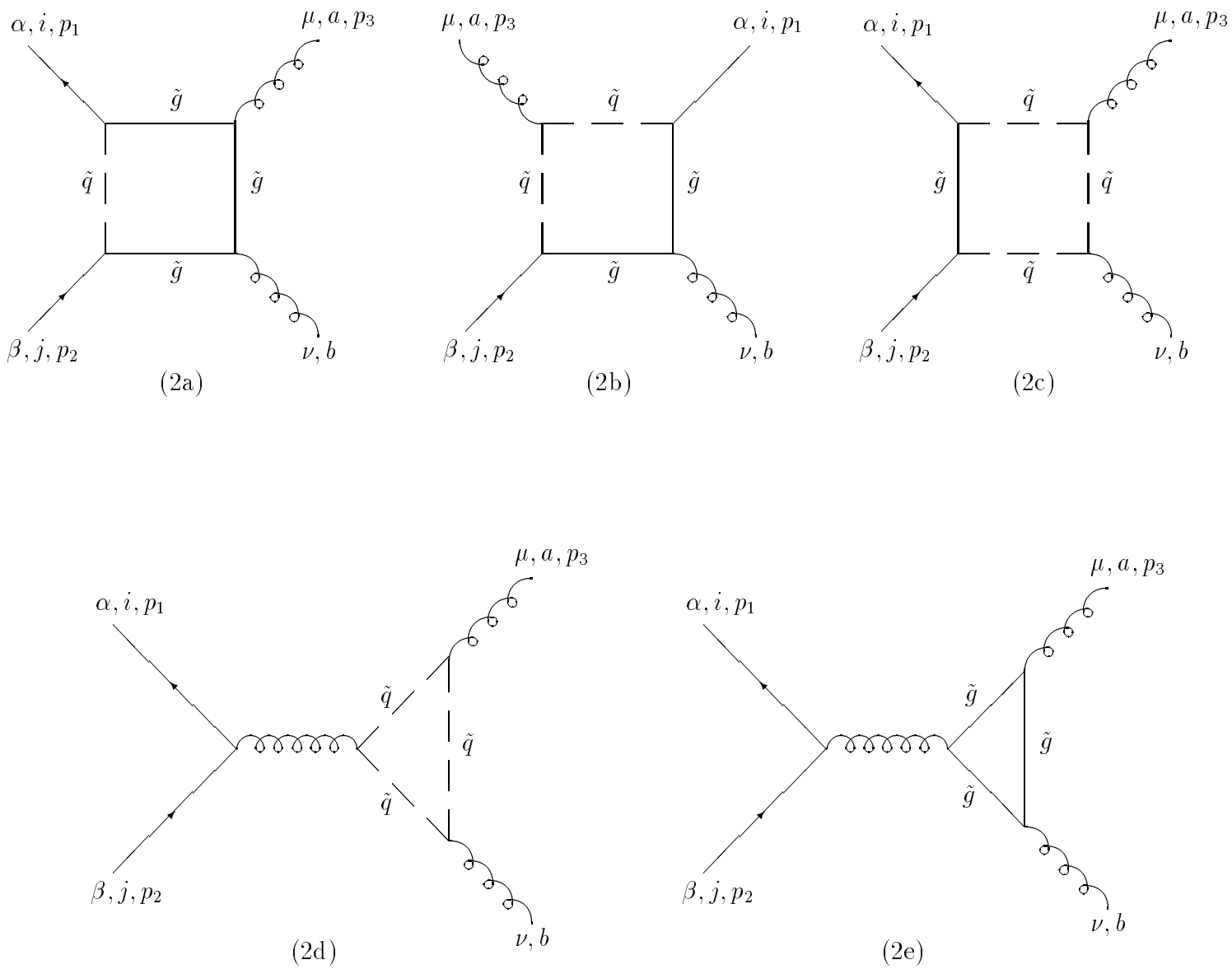


Fig. 2